Journal of Engineering Physics and Thermophysics, Vol. 76, No. 2, 2003

HEAT-SOURCE-INITIATED THERMOELASTIC STATE OF A SEMIINFINITE PLATE WITH AN EDGE CRACK

S. Ya. Matysyak,[a] **A. A. Evtushenko,**[b] **and V. M. Zelenyak**[c]

UDC 539.377

A study is made of the influence of the internal heat source on the factors of intensity of the stresses at the tip of an arbitrarily oriented edge cut in a homogeneous half-plane.

Introduction. Heating of materials by locally distributed heat sources is widely used in engineering practice. In the presence of defects of the type of cracks, inclusions, etc., in thermoelastic bodies, one observes considerable singularities of the temperature stresses near the tips of the defects, which often lead to failure of structures. The temperature stresses initiated by the heat source near the internal tip of an edge crack which is perpendicular to the boundary of a half-plane have been investigated earlier in [1].

In the present work, we consider the problem of static thermoelasticity for a semiinfinite plate with an arbitrarily oriented edge crack. It is assumed that the surfaces of the plate and the edges of the crack are free of external forces and are heat-insulated. The thermal stresses in the body are caused by an internal stationary heat source acting near the tip of the cut. As a consequence of the linearity of the thermoelasticity problem we represent its solution in the form of a sum of the solutions of:

(1) the basic problem of thermoelasticity for a semiinfinite plate without a crack in the case of the action of a heat source;

(2) the perturbed problem of elasticity theory for a plate with an edge crack to whose edges one applies forces (of opposite sign) obtained by solution of the first problem;

(3) the problem on determination of the stressed state initiated by the perturbation of the temperature field as a consequence of the presence of the crack.

In this work, we consider only the first and second problems, since the third problem has already been solved in [2]. A singular integral equation for the derivative of the jump of normal displacements on the cut edges has been obtained. The numerical solution of this equation has been constructed by the mechanical-quadrature method [3]. The influence of the angle of orientation of the crack on the factors of intensity of normal K_{I} and tangential K_{II} stresses has been investigated.

Formulation of the Problem. Let us consider a homogeneous isotropic semiinfinite plate (generalized plane stressed state) having an arbitrarily oriented edge crack. A lumped heat source of constant power is acting near the internal tip of the crack (Fig. 1). The lateral surfaces of the plate and the edges of the crack are heat-insulated. We disregard a possible contact of the cut edges. The temperature and the stress decrease with distance from the plate surface.

In the rectangular coordinate system xOy (Fig. 1), the solution of the first (basic) problem has the form [4]

$$\sigma_{xx}(x, y) = Q\beta_t \left[\ln \frac{p_2}{p_1} - \frac{4h\,(y-h)}{p_1^2} + N(x, y) \right],$$

[a]Warsaw University, Warsaw, Poland; [b]Ya. S. Podstrigach Institute of Applied Problems of Mechanics and Mathematics, National Academy of Sciences of Ukraine, L'vov, Ukraine; Polytechnic Institute, Łódź, Poland; [c]"L'vovskaya Politekhnika" Technical University, L'vov, Ukraine; email: dept13@iapmm.lviv.ua. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 76, No. 2, pp. 134–137, March–April, 2003. Original article submitted May 23, 2002; revision submitted August 26, 2002.

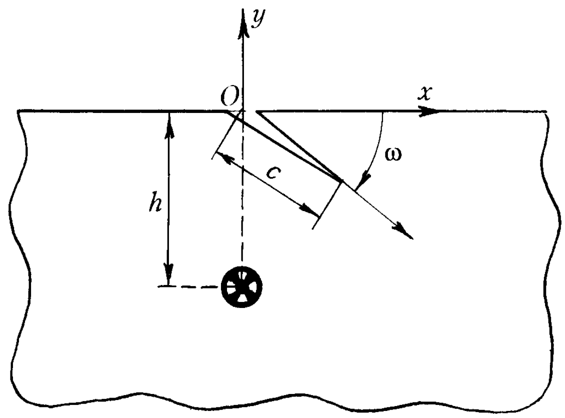

Fig. 1. Scheme of the problem.

$$\sigma_{yy}(x, y) = Q\beta_t \left[\ln \frac{p_2}{p_1} - N(x, y) \right],$$
$$\sigma_{xy}(x, y) = \frac{4Q\beta_t\, hyx}{p_1^2} \left[\frac{y-h}{p_1^2} - \frac{y+h}{p_2^2} \right], \tag{1}$$

where

$$N(x, y) = \frac{4hyx^2}{p_1^2} \left(\frac{1}{p_1^2} - \frac{1}{p_2^2} \right) - \frac{2hy}{p_1^2};$$
$$p_1^2 = x^2 + (y-h)^2;$$
$$p_2^2 = x^2 + (y+h)^2; \tag{2}$$
$$Q = \frac{q}{4\pi\lambda d}.$$

Solution of the second (perturbed) problem by the methods of the theory of functions of a complex variable [1] is reduced to the solution of the singular integral equation

$$\int_{-1}^{1} [K(\xi, \eta)\, g'(\xi) + L(\xi, \eta)\, \overline{g'(\xi)}]\, d\xi = \pi P(\eta), \quad |\eta| < 1, \tag{3}$$

where

$$K(\xi, \eta) = \frac{1}{\xi - \eta} + \frac{1}{2} \left\{ \frac{1}{1 + \eta - (1+\xi)\exp(2i\omega)} + \right.$$
$$+ \frac{(1+\eta)^2 + (1+\xi)(1+\eta)[1 - 4\exp(-2i\omega) + \exp(-4i\omega)]}{[1 + \eta - (1+\xi)\exp(-2i\omega)]^3} +$$
$$\left. + \frac{(1+\xi)^2 [\exp(-2i\omega) - \exp(-4i\omega) + \exp(-6i\omega)]}{[1 + \eta - (1+\xi)\exp(-2i\omega)]^3} \right\}; \tag{4}$$

$$L(\xi, \eta) = \frac{1+\xi}{2}\left\{\frac{1-\exp(-2i\omega)}{[1+\eta-(1+\xi)\exp(-2i\omega)]^2} + \frac{\exp(2i\omega)[1-\exp(2i\omega)]}{[1+\eta-(1+\xi)\exp(2i\omega)]^2}\right\}; \quad (5)$$

$$P(\eta) = \sigma + i\tau ; \quad (6)$$

$$\eta = x_1/c , \quad x_1 = x\cos\omega - y\sin\omega - c/2 . \quad (7)$$

The vinculum denotes the conjugates. The sought function $g'(\xi)$ in the integral equation (3) represents the derivative of the jump of normal displacements of the crack edges.

Numerical Analysis. In the problem in question, the mechanical-quadrature method of solution of the singular integral equation used in the case of internal cracks [5] cannot be employed directly: the kernels $K(\xi, \eta)$ (4) and $L(\xi, \eta)$ (5) of the integral equation (3) contain an "immovable" singularity at $\xi = \eta = -1$ in addition to a singularity of the Cauchy type. Therefore, we use the method (proposed in [3]) of numerical solution of a singular integral equation with an "immovable" singularity, which enables us to seek the solutions in the class of functions of index 1 (i.e., functions processing an integrable singularity in the cut tips). The function $g'(\xi)$ is sought in the form

$$g'(\xi) = \frac{V(\xi)}{\sqrt{1-\xi^2}} , \quad |\xi| < 1 , \quad (8)$$

where $V(\xi)$ is the Lagrange interpolation polynomial

$$V(\xi) = \frac{2}{\pi} \sum_{k=0}^{n-1}{}' \sum_{m=1}^{n} \cos\left(\frac{2m-1}{2n} k\pi\right) V(\xi_m)\, T_k(\xi) , \quad (9)$$

by the Chebyshev nodes

$$\xi_m = \cos\frac{\pi(2k-1)}{2n} , \quad k = 1, 2, ..., n . \quad (10)$$

Here $\sum'$ implies that the first term under the sign of summation should be multiplied by 1/2.

Since the function $V(\xi)$ at $\xi = -1$ (i.e., in the tip of the crack emerging at the edge of the plate) has a singularity of order lower than 1/2, we assume that

$$V(-1) = 0 . \quad (11)$$

With account for relations (8)–(10), the singular integral equation (3)–(7) and equality (11) will be represented in the form of the system of linear algebraic equations

$$\frac{1}{n}\sum_{k=1}^{n} [K(\xi_k, \eta_k)\, V(\xi_k) + L(\xi_k, \eta_m)\, \overline{V}(\xi_k)] = P(\eta_m) ,$$

$$\sum_{k=1}^{n} (-1)^k\, V(\xi_k) \tan\left(\frac{2k-1}{4n}\pi\right) = 0 , \quad \eta_m = \cos\frac{\pi m}{n} , \quad m = 1, 2, ..., n-1 . \quad (12)$$

After the solution of (12) for the values of $V(\xi_k)$ according to the formula [6]

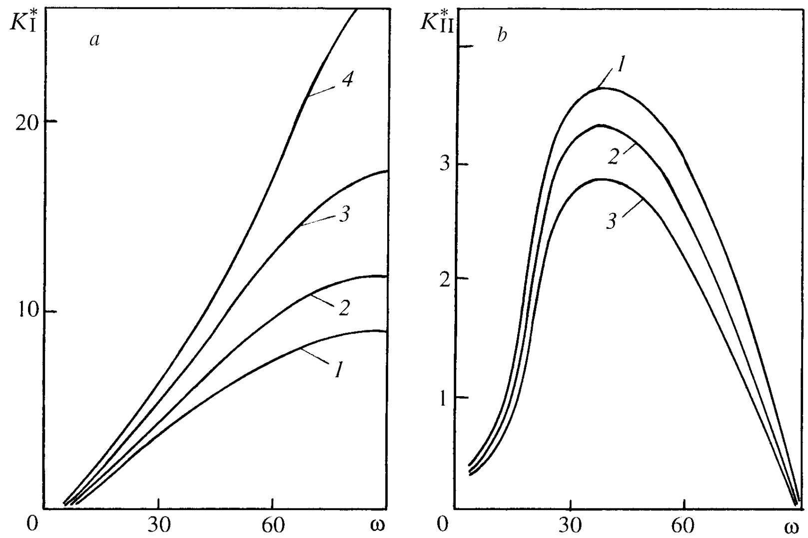

Fig. 2. Dimensionless stress intensity factors K_I^* (a) and K_{II}^* (b) vs. slope of the crack ω: 1) $c^* = 0.1$, 2) 0.3, 3) 0.5, and 4) 0.7.

$$K_I - iK_{II} = \sqrt{\frac{c}{2}}\,\frac{1}{n}\sum_{k=1}^{n}(-1)^k V(\xi_k)\,\mathrm{ctan}\left(\frac{2k-1}{4n}\pi\right)$$

we determine the stress intensity factors near the internal tip of the crack.

The independent input parameters of the problem are $c^* = c/h$ and ω, We needed no more than 20 collocation nodes to attain a relative accuracy of computations of 1%.

The dependences of the dimensionless stress intensity factors

$$K_I^* = \frac{K_I}{Q\beta_t\sqrt{c}}, \quad K_{II}^* = \frac{K_{II}}{Q\beta_t\sqrt{c}}$$

on the slope of the cracks ω are presented in Fig. 2.

CONCLUSIONS

It has been established that:

(1) the intensity of normal stresses is much higher than the intensity of tangential stresses ($K_I^* >> K_{II}^*$);

(2) irrespective of the crack length, the maximum value of the normal stresses is attained when the crack is perpendicular to the plate's edge ($\omega \approx 90^o$), while that of the tangential stresses is attained at $\omega \approx 35^o$;

(3) as the heat source approaches the internal tip of the crack, the intensity of the normal stresses increases while the intensity of the tangential stresses decreases.

NOTATION

c, crack length; d, plate thickness; h, distance from the plate's edge to the heat source; x and y, axes of the rectangular coordinate system; ω, slope of the crack; $\beta_t = \alpha_t E$; E, Young modulus; λ, thermal conductivity; α_t, coefficient of linear thermal expansion; q, heat-source power; K_I and K_{II}, stress-intensity factors; σ_{xx}, σ_{yy}, and σ_{xy}, stresses; σ and τ, normal and tangential stresses at the edges of the cut; T_k, Chebyshev polynomials of the first kind of order k; c^*, dimensionless crack length; $i = \sqrt{-1}$; ξ, variable of integration in the interval [–1; 1]. Subscripts: t, thermal; I and II, refer to the normal and tangential stresses.

REFERENCES

1. V. M. Zelenyak and L. L. Libatskii, *Vestn. GU "L'vovskaya Politekhnika," Differents. Uravneniya*, No. 286, 56–62 (1995).
2. S. Ya. Matysyak, A. A. Evtushenko, and V. M. Zelenyak, *Mat. Metody Fiz.-Mekh. Polya*, **43**, No. 2, 127–134 (2000).
3. M. P. Savruk, *Two-Dimensional Problems of Elasticity for Bodies with Cracks* [in Russian], Kiev (1981).
4. D. B. Grilitskii and M. P. Osiv, *Heat Conduction and Elasticity Problems for Plates* [in Russian], L'vov (1974).
5. V. V. Panasyuk, M. P. Savruk, and A. P. Datsyshin, *Distribution of Stresses near Cracks in Plates and Shells* [in Russian], Kiev (1976).
6. S. Krenk, *Quart. Appl. Math.*, **32**, No. 1, 479–484 (1975).